

## Improvements in the precision of betaray spectroscopy

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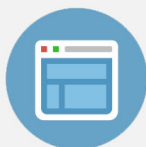
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The beta-ray spectrometer discussed in the following pages has recently been completed at the California Institute of Technology. It is shown in cross section on the cover.

# Improvements in the Precision of BETA-RAY

By Jesse W. M. DuM

THE DIRECT MEASUREMENT of gamma-rays only yields about half the picture in the study of nuclear energy levels. The other, and indeed to most physicists more familiar, half concerns the  $\beta$ -rays, including in this term both the continuous  $\beta$ -ray spectrum and the line spectrum by "conversion", either internal or external. Ever since 1948, therefore, we have been much occupied with the design and construction along rather novel lines<sup>1</sup> of a high precision helical focusing magnetic  $\beta$ -ray spectrometer planned as a companion instrument to the crystal diffraction spectrometer as regards absolute precision and accuracy. We have only very recently completed this instrument and made the first tests on it which indicate that it will meet all our expectations both as to high absolute accuracy and high luminosity and sensitivity to weak sources.

The original choice of a homogeneous magnetic field for the instrument was made because such a field lends itself most easily to the analysis for optimum conditions of high luminosity and high resolving power. Other advantages coming from this choice have appeared, however, in the course of the design. For example, it was decided at an early stage to measure and stabilize the field by means of proton resonance and it turns out that the rather weak fields of about 30 gauss needed at low  $\beta$ -ray energies are much easier to measure and stabilize by this method if they are homogeneous, because this permits the use of a large sample volume and therefore affords a better signal-to-noise ratio.

To avoid the well-known troubles and uncertainties from hysteresis effects, it was decided to make the instrument entirely free of any ferromagnetic material. The reinforcing iron in the concrete floor and ceiling of the building has been kept about five feet distant and oriented with the long axis at right angles to the dipole set up by the field of the instrument. We have not been able to detect any harmful effects from this reinforcing iron.

In a homogeneous magnetic field (with electric fields absent) the path of a  $\beta$ -particle is a uniform helix. A family of such helices, identical save for the azimuth angle at which they are emitted from the source, lies on the surface of a figure of revolution with a sinusoid

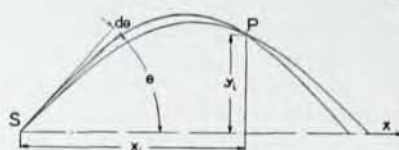


Fig. 1. Two sinusoidal electron traces, for the same kinetic energy and homogeneous field intensity, but with slightly different colatitude angles of take-off, determine a point of intersection  $P$  which fixes the location of the annular slit in the  $\beta$ -ray spectrometer.

Fig. 2. The kinetic energy  $\epsilon$  of the "ultimate" is at a parabolic minimum among the energy of the entire family of traces from  $S$  through  $P$ . The ultimate trace, the limit of the coalescing whose intersection located  $P$ , appears shaded in the upper diagram.

for its profile, as can be very easily proved. For the purposes of spectral resolution, the present instrument makes use only of the coordinates  $r$  and  $Z$ , the radial distance of the electron from the axis of the instrument and its axial distance from the source. The azimuth,  $\phi$ , in these cylindrical coordinates is unimportant. We shall therefore refer to the sinusoidal trace or profile of the figure of revolution on which the  $\beta$ -ray trajectory lies simply as the trace or trajectory of the electron.

Fig. 1 shows two sinusoidal electron traces of the same kinetic energy with slightly different colatitude angles of take-off,  $\theta$ , at the source. These two traces have a point of intersection,  $P$ . If we diminish the difference in  $\theta$  until the two traces coalesce we can find by very simple analysis the limiting position of  $P$  for the ultimate intersection. The angle  $\theta_1$  of the coalescing pair completely determines the ratio of the coordinates  $x_1$  and  $y_1$  of the ultimate intersection point  $P$ . This point, it should be fairly clear, is the place to put the annular resolving slit. At other fields or other kinetic energies the ultimate intersection points  $P$  for the same take-off angle  $\theta_1$  will lie on a right circular conical surface with apex at  $S$  through the point  $P$ .

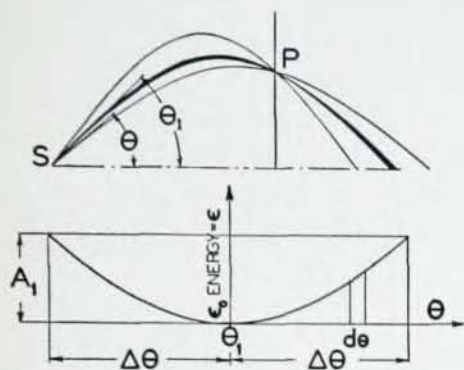
Now, referring to Fig. 2, if we inquire how the kinetic energies of electrons with traces starting from  $S$  and passing through  $P$ , but with take-off angles  $\theta$  different from  $\theta_1$ , compare with the kinetic energy of the electrons of the ultimate trace,  $\theta_1$ , we find that the ultimate trace has a kinetic energy which is at a parabolic minimum of the entire family through  $S$  and  $P$ .

We see from this that the choice of an ultimate in-

<sup>1</sup> The instrument here described has been designed following a theory of the writer developed in an earlier paper, "Conditions for Optimum Luminosity and Energy Resolution in an Axial  $\beta$ -Ray Spectrometer with Homogeneous Magnetic Field," *Rev. Sci. Instr.* 20, 160 (1949). All the capital expenditures for the design and construction of this instrument have been derived from three grants to the California Institute of Technology kindly made available for this purpose by the Research Corporation. A more complete technical description of the instrument has been prepared in the form of Special Technical Reports to the Research Corporation and to the Office of Naval Research respectively, the latter numbered, S.T.R. No. 16.



# SPECTROSCOPY



$$\frac{d|p|}{|p|} = B(\theta_1)(d\omega)^2; B(\theta_1) = 6 + 6\cot^2\theta_1 + 2\psi_1^2 \tan^2\theta_1.$$

$$\frac{d\epsilon}{\epsilon} = \frac{\epsilon + 2}{\epsilon + 1} B(\theta_1)(d\omega)^2. \quad \epsilon = \frac{h\nu}{m_0c^2}, \quad -\psi_1 \tan^2\theta_1 = \tan\psi_1.$$

tersection point for the location of the annular slit makes it possible to have the inhomogeneity in kinetic energy corresponding to a given utilized range of take-off angles such that the former is one order of infinitesimal smaller than the latter, as shown on the figure, such that the relative inhomogeneity in the absolute value of the  $\beta$ -ray momentum,  $d|p|/|p|$ , can be expressed as a function, say  $B(\theta_1)$  of  $\theta_1$  multiplied by the square of the utilized fraction of the sphere  $(d\omega)^2$  surrounding the source. We find that  $B(\theta_1)$  has a minimum at about the point  $\theta_1 = 45^\circ$ .

This means that in the vicinity of  $\theta_1 = 45^\circ$  we can use the largest solid angle about the source and pay the smallest price in momentum or energy inhomogeneity in return for it. No very great sacrifice is made over the entire range from  $35^\circ$  to  $55^\circ$  but if we go to angles as small as  $15^\circ$ , not at all uncommon in many axial  $\beta$ -ray spectrometers, we see that the inhomogeneity in energy for a given solid angular aperture is some four times as great as at  $45^\circ$ . This sacrifice has, we believe, been unwittingly made by designers of  $\beta$ -ray spectrometers in the past because of the attractive but superficial analogy to optical systems with nearly paraxial rays and because the extremely complicated expansions in infinite series to express the trajectories has made the solution for an optimum in such cases hopelessly complicated. We think that all such designs suffer from being too long and slim.

Let us now turn to a consideration of the various causes of finite spectral line breadth in our spectrometer. Fig. 3 shows the line profiles from four of these

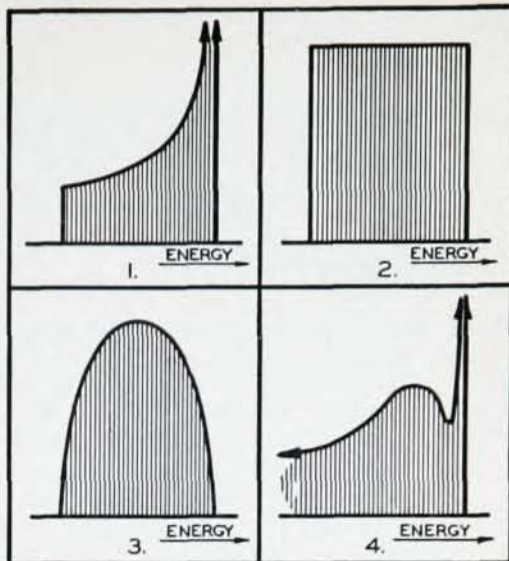


Fig. 3. Shapes of three component line profiles (1) from spherical aberration, (2) from the finite opening of the main annular resolving slit, (3) from the finite size of a disc-shaped source originally conceived as constituting, when folded together, the characteristic instrumental line profile, if the retardation of some of the  $\beta$ -rays in the source can be neglected. At (4) is shown a fourth component, somewhat idealized, which we believe represents a typical distribution of  $\beta$ -ray population over energy (for an initially homogeneous energy class) when retardation in the source material cannot be neglected. The debatable features of this fourth profile concern the way (shape) in which it trails off on the low energy side but its important feature in the present application (which we believe is indisputable) is the first order discontinuity at the high-energy limit of the curve.

causes, each profile being the idealized shape which would be obtained if all the other causes were negligible. The actual profile is then the fold or composition of these four different profiles into each other. The first profile, the one due to finite aperture angle, has an infinite discontinuity on its high-energy edge and decays toward lower energies according to an inverse half-power law. The second profile, coming from finite opening of the annular resolving slit, is simply a rectangle. The third profile, from the finite diameter of a disc-shaped source, has an elliptical or semicircular profile, and the fourth profile, the inhomogeneity introduced into an initially monokinetic group of  $\beta$ -rays by retardation from back-scattering and passage through source material, like the first, has an infinite discontinuity on its high energy edge. The fold of these four profiles into each other, when their breadths are matched so as to minimize the resulting breadth at half maximum height, yields a line profile which has unfortunately no well-marked fiducial features to which absolute significance can be attached.

Dr. D. E. Muller, looking at these curves recently, noticed that all of them save the one from the finite disc source, have discontinuities on the high energy edge and two of these three have infinite discontinuities there. He reasoned that if one could so modify the shape of the source as to yield a profile for it which

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would also have an infinite discontinuity on the high energy edge, then the fold of all four profiles must exhibit a very clearly marked fiducial feature on its high energy boundary. This led him to the ingenious idea of making a source in the shape of a convex cone as shown in Fig. 4.

The semi-apex angle of the cone is made equal to the slope of the ultimate  $\beta$ -ray trace where the latter passes over the inner edge of the annular resolving slit. This inner edge must be situated at the ultimate intersection point  $P$  for the ultimate trace. As the magnetic field is gradually increased in a spectral exploration, the very last atoms on the surface of the conical source which can emit  $\beta$ -rays able to surmount the inner edge of the slit at one point of its circumference are those which lie along an entire element of the cone, a vanishingly narrow sector of the conical surface but representing a much greater area, however, than the vanishingly small segment which constitutes the corresponding case on a disc source. Dr. Muller has shown that the profile corresponding to the conical source is the one shown in the insert in Fig. 4. This profile has the desired infinite discontinuity on the high energy edge and it has an inverse half power law of decay toward lower energies in this vicinity. Thus, with the conical source, all four chief contributions to the line structure have abrupt vertical discontinuities on their high energy boundaries. Only one of these, the rectangle from the finite annular slit, has a finite ordinate at this edge. Because of these facts the resultant fold of the four profiles, while it will not exhibit an abrupt first order discontinuity at its high energy boundary, should exhibit a well marked second order discontinuity there.

Fig. 5 shows the high energy edge of one of the actual line profiles observed with our spectrometer using a conical source. The high energy edge of the line is characterized by a steep linear decline which lends itself well to the precise location of the above-mentioned second order discontinuity (which we shall henceforth refer to as the  $Q$ -point). This discontinuity is masked somewhat by a small residual fillet which extends over an energy range of only a few parts in ten thousand. This probably is partly accounted for by the natural breadth of the x-ray level from which the conversion electron originates and partly by small residual imperfections in our instrument.

The great advantage of this  $Q$ -point on the profile is that it has a clearly defined significance. The magnetic field intensity at which the  $Q$ -point is observed to occur is to be associated with the radius of curvature of the helical  $\beta$ -ray trajectory for the ultimate ray. This in turn can be computed with high precision from a knowledge of the exact location of the inner slit jaw, in radial and axial distance, from the point of the conical source. We believe that this is the first time in the history of such instruments that an absolute significance such as this can be securely attached to some well defined feature of the line profile. This is another, and very important, advantage resulting from the adoption of a homogeneous magnetic field.

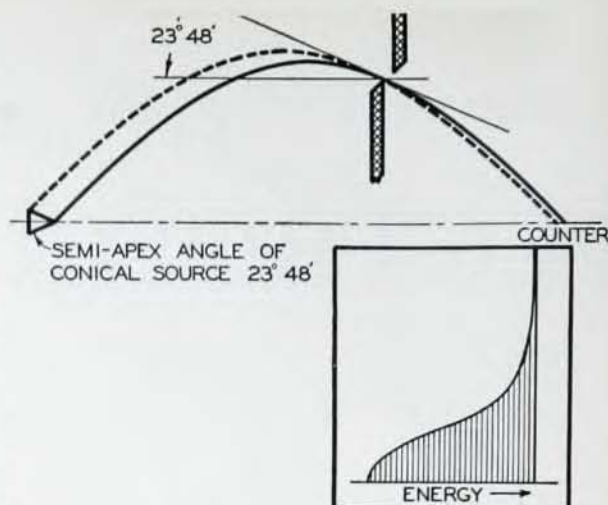


Fig. 4. Geometry determining the semi-apex angle of the Muller-type conical sources. Size of cone is of course greatly exaggerated. Insert below shows calculated shape of component line profile contributed by such a conical source. This is the profile that replaces the semi-elliptical profile (3) in Fig. 3 when conical instead of disc-shaped source is used.

Fig. 6 shows a sample spectrum made with the spectrometer with a thorium (Th B + C + C'') source of about  $\frac{1}{4}$  mc strength, electro-deposited on an aluminum cone of 2.5 mm base diameter. The  $F$  and  $I$  lines (Ellis notation) are about 0.8 percent wide at half maximum height. The points shown represent readings for 32 second intervals. At the peak of the  $F$  line 35,200 counts were recorded in that interval and 3,200 counts in the adjacent continuous  $\beta$ -spectrum in the same interval. By reducing the size of the conical source to 1 mm base diameter and stopping down the take-off angle, and the annular resolving slit to match, we obtained the  $F$ -line profile shown in the insert in Fig. 6, which is only 0.35 percent wide at half maximum height. The counting interval here was two minutes, and the solid angle utilized around the source about one percent of the total sphere. Finally, as a test of what can be done in measuring line energies by the  $Q$ -point we show Table 1. Here we have the results of six inde-

Table 1. Data on Reproducibility of Measurements of Thorium  $F$  and  $I$  Lines with Homogeneous Field  $\beta$ -Spectrometer (L. Bogart, J. Kohl and J. Wilts)

Run No.	Date	F-Line $Q$ -Point	I-Line $Q$ -Point
1	27 March	418.42 kc	528.65 kc
2	28 March	418.42 kc	528.59 kc
3	31 March A.M.	418.48 kc	528.67 kc
4	31 March P.M.	418.38 kc	528.60 kc
5	1 April A.M.	418.43 kc	528.58 kc
6	1 April P.M.	418.40 kc	528.58 kc
Line	Mean $Q$ -Point $\sigma$ (Single Observation) $\sigma$ (of the mean)		
$F$	418.442 kc	$\pm 0.034$ kc	$\pm 0.014$ kc
$I$	528.612 kc	$\pm 0.039$ kc	$\pm 0.018$ kc
$\frac{(H\rho)_I}{(H\rho)_F} = \begin{cases} 1.26329 \pm 0.00006 \text{ (Std. Dev.)} & \text{(Our value computed from above)} \\ 1.26317 \pm 0.00012 \text{ (P.E.)} & \text{(H. Craig, Phys. Rev. 85, 688 (1952))} \\ 1.26319 \pm 0.00005 \text{ (P.E.)} & \text{(G. Lindstrom, Phys. Rev. 83, 465 (1952))} \end{cases} $			

Note: Our values are subject to slight revision for possible systematic errors in our frequency meter.



pendent runs to determine the  $Q$ -point for the  $F$ -line and for the  $I$ -line and the mean value of each together with the standard deviation of the mean of the six and of a single observation. This last is seen to be substantially smaller than one part in 10,000 in both cases. We also show, for comparison, our ratio of  $H\rho$ 's for the  $I$  and  $F$  lines and two recent precision determinations made with iron spectrometers, one at Harvard and one at the Nobel Institute in Stockholm. We do not offer these results as definitive, however, because the proton resonance frequency was measured with an inferior (war surplus) frequency meter, the only one available until we receive delivery of a Hewlett-Packard frequency scalar.

Fig. 5 shows one of the curves of the linear high-energy edge of the  $F$ -line and of the adjacent background by means of which the  $Q$ -point shown was determined. The spread or uncertainty of  $\pm 0.039$  kc shown in the plot of Fig. 5 is the standard deviation of a single observation as computed from the consistency of the six measurements shown in Table I. It is seen to be comparable with the internal consistency to be expected from the statistical counting uncertainties in the profile and background.

Full details with photographs of the design and construction of this new instrument, including the proton resonance stabilization of the field to one part in  $10^4$ , have been given in recent Special Technical Reports to the Office of Naval Research and to the Research Corporation.<sup>2</sup> They will also be published in a forthcoming paper. The homogeneous field is set up by an ellipsoidal oil-immersed water cooled field winding consisting of 40 coils of graduated diameters, spaced so as to have as nearly as possible the same number of ampere turns per unit length of major diameter everywhere over its entire length. The range of field intensities from 30 gauss to 1300 gauss corresponds to a range of  $\beta$ -ray energies from about 20 Kev to 5 Mev.

The source can be introduced through an air-lock and provision by means of a sylphon bellows and micrometer screws is made to adjust minutely the centering of the source after it is in the instrument. Slit jaws are

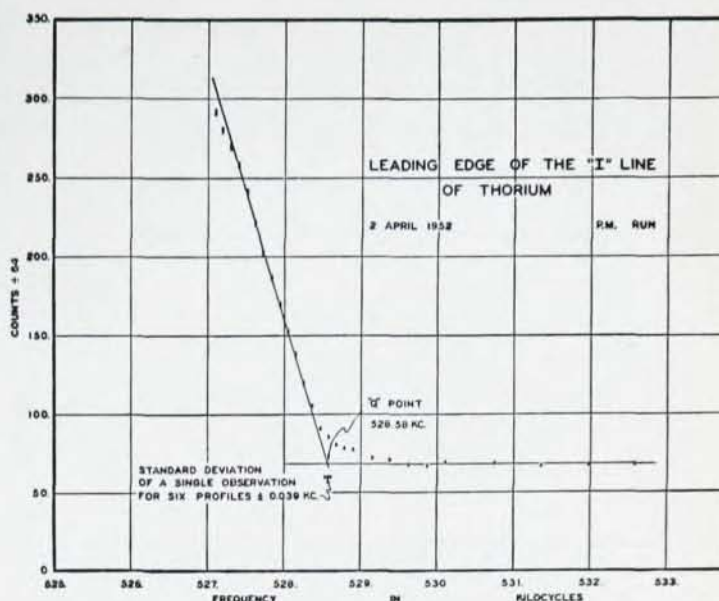


Fig. 5. Typical high-energy edge of the "I" line of thorium (Ellis' notation) as used in determining the "Q"-point for the data of Table IV.

provided for controlling the maximum and minimum take-off angles at the source, and these can be adjusted through calibrated mechanical controls after vacuum has been established. A similar calibrated control permits micrometric adjustment of the annular resolving slit under vacuum. Finally, there is provided a shutter which through mechanical controls under vacuum operation can be used to isolate a small segment in any desired azimuth of the annular resolving slit, and to block the remainder. This is of great value in centering the source with minute precision so that the  $\beta$ -rays focus in perfectly concentric register all around the annular slit.

<sup>2</sup> An Axial Focusing Magnetic  $\beta$ -Ray Spectrometer of High Luminosity, Resolving Power and Precision with Proton-Resonance-Stabilized Homogeneous Field Without Iron. J. W. M. DuMond, L. Bogart, J. L. Kohl, D. E. Muller, and J. R. Wilts, Special Technical Report No. 16, Contract N6onr-244, T.O. IV.

Fig. 6. Spectrum of a conical electrodeposited thorium (B + C + C'') source of about 0.25 mc strength. Base diameter of cone 2.5 mm. The abscissae are readings on an arbitrary scale of divisions of the fine adjustment dial of the oscillator whose frequency, through the proton resonance heads and automatic field stabilizing system, fixes the magnetic field intensity of the spectrometer. Points on this scale corresponding to 100 kev and 250 kev  $\beta$ -ray energy are marked. In the insert at upper left is shown the profile of Th E and F lines (Ellis' notation) with higher resolution. This was made with a conical source of base diameter 1.0 mm and with other instrument parameters matched. The decline in the continuous spectrum toward lower energies is the cut-off effect of the 0.5 mil mica window used on the counter.

